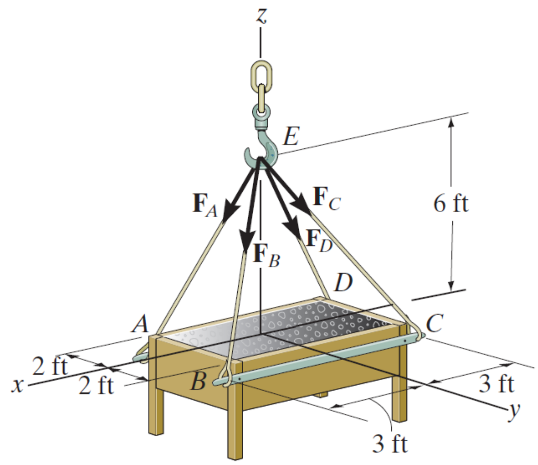
Problem 2-105

If the resultant of the four forces is $\mathbf{F}_R = \{-360\mathbf{k}\}$ lb, determine the tension developed in each cable. Due to symmetry, the tension in the four cables is the same.



Probs. 2–104/105

Solution

Write the position vectors to the points A, B, C, D, and E.

$$\mathbf{r}_A = \langle 3, -2, 0\rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 3, 2, 0\rangle \text{ ft}$$

$$\mathbf{r}_C = \langle -3, 2, 0\rangle \text{ ft}$$

$$\mathbf{r}_D = \langle -3, -2, 0\rangle \text{ ft}$$

$$\mathbf{r}_E = \langle 0, 0, 6\rangle \text{ ft}$$

The position vector going from E to A is then

$$\begin{aligned}\mathbf{r}_{EA} &= \mathbf{r}_A - \mathbf{r}_E \\ &= \langle 3, -2, -6\rangle \text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{EA}| &= \sqrt{(3)^2 + (-2)^2 + (-6)^2} \text{ ft} \\ &= 7 \text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{EA}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{EA} = \frac{\mathbf{r}_{EA}}{|\mathbf{r}_{EA}|} = \frac{\langle 3, -2, -6\rangle}{7}$$

The force $\mathbf{F}_A$ can now be written.

$$\mathbf{F}_A = F_A \hat{\mathbf{u}}_{EA} = F\,\frac{\langle 3, -2, -6\rangle}{7}$$

On the other hand, the position vector going from E to B is

$$\begin{aligned}\mathbf{r}_{EB} &= \mathbf{r}_B - \mathbf{r}_E \\ &= \langle 3, 2, -6\rangle \text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{EB}| &= \sqrt{(3)^2 + (2)^2 + (-6)^2} \text{ ft} \\ &= 7 \text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{EB}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{EB} = \frac{\mathbf{r}_{EB}}{|\mathbf{r}_{EB}|} = \frac{\langle 3, 2, -6\rangle}{7}$$

The force $\mathbf{F}_B$ can now be written.

$$\mathbf{F}_B = F_B\hat{\mathbf{u}}_{EB} = F\,\frac{\langle 3, 2, -6\rangle}{7}$$

On the other hand, the position vector going from E to C is

$$\begin{aligned}\mathbf{r}_{EC} &= \mathbf{r}_C - \mathbf{r}_E \\ &= \langle -3, 2, -6\rangle \text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{EC}| &= \sqrt{(-3)^2 + (2)^2 + (-6)^2} \text{ ft} \\ &= 7 \text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{EC}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{EC} = \frac{\mathbf{r}_{EC}}{|\mathbf{r}_{EC}|} = \frac{\langle -3, 2, -6\rangle}{7}$$

The force $\mathbf{F}_C$ can now be written.

$$\mathbf{F}_C = F_C\hat{\mathbf{u}}_{EC} = F\,\frac{\langle -3, 2, -6\rangle}{7}$$

On the other hand, the position vector going from E to D is

$$\begin{aligned}\mathbf{r}_{ED} &= \mathbf{r}_D - \mathbf{r}_E \\ &= \langle -3, -2, -6\rangle \text{ ft.}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{ED}| &= \sqrt{(-3)^2 + (-2)^2 + (-6)^2} \text{ ft} \\ &= 7 \text{ ft.}\end{aligned}$$

Divide $\mathbf{r}_{ED}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{ED} = \frac{\mathbf{r}_{ED}}{|\mathbf{r}_{ED}|} = \frac{\langle -3, 2, -6 \rangle}{7}$$

The force $\mathbf{F}_D$ can now be written.

$$\mathbf{F}_D = F_D \hat{\mathbf{u}}_{ED} = F\,\frac{\langle -3, -2, -6 \rangle}{7}$$

Add the four forces to get their resultant.

$$\begin{aligned}
\mathbf{F}_R &= \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D \\
\langle 0, 0, -360 \rangle \text{ lb} &= F\,\frac{\langle 3, -2, -6 \rangle}{7} + F\,\frac{\langle 3, 2, -6 \rangle}{7} + F\,\frac{\langle -3, 2, -6 \rangle}{7} + F\,\frac{\langle -3, -2, -6 \rangle}{7} \\
&= \left\langle 0, 0, -\frac{24F}{7} \right\rangle
\end{aligned}$$

Match the components.

$$-360 \text{ lb} = -\frac{24F}{7}$$

Therefore, the tension in each cable is

$$F = 105 \text{ lb}.$$